Centripetal Force - Inquiry

OBJECTIVE

To verify that a mass moving in circular motion experiences a force directed toward the center of its circular path. To determine how the mass, velocity, and radius affect a particle's centripetal force. To explain why the centripetal force is necessary for circular motion.

INTRODUCTION

When any mass **m** moves in a circle of radius **r** with a constant velocity **v**, it experiences a centripetal force directed toward the center of its circular path. For this laboratory, we will study the case of uniform circular motion, where the speed (a scalar quantity) of the moving mass is held constant. Yet, as the mass moves in a circular path, the velocity (a vector quantity) is constantly changing. By definition a velocity that changes over time is acceleration. Thus, this change in velocity comes from a centripetal acceleration due to a centripetal force. The centripetal force is that force that pulls an object out of a straight-line path and into a circular path. In this laboratory, the centripetal force will be provided by gravity and a spring (in separate cases). By measuring the frequency of rotation, the object's mass, path radius and the spring force, the centripetal force can be experimentally determined.

APPARATUS

(1) spring centripetal force apparatus, (1) meter stick, (1) slotted mass set, (1) stopwatch, (1) balance, (1) string & mass centripetal force apparatus, and (10) nuts.

THEORY

As previously stated, when a particle moves in a circular path, its velocity is constantly changing. The velocity of the particle is tangent to the path of its motion at all times. Thus, as it is ever changing its path, it is ever changing its velocity. The magnitude of the velocity, however, will remain constant. Note, the speed (velocity magnitude) does not have to be constant but would present further study to pursue what amounts to a changing centripetal force.

The figure below illustrates this motion.

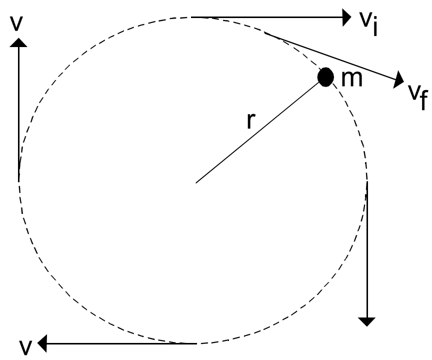

Figure 1

Note, if the mass were to break from the confines of its circular path, it would move off in a direction that is tangent to the circle at the point of release. Two of the tangential velocities have been intentionally labeled v_i and v_f to illustrate the direction of the resultant velocity. This is done by applying the relation for change in velocity:
$\Delta v = v_f - v_i$. From the two labeled vectors in Figure 1 and the previously cited equation, we get:

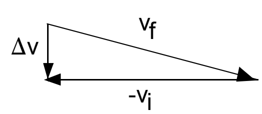

Figure 2

Note that Δv points toward the center of the circular path. This vector direction, for the velocity, will always be the same as long as we take v_i and v_f to have a very short time interval between them. This change in velocity results from a centripetal acceleration. It can be further shown that, the magnitude of this centripetal acceleration is given by:

$$a_c = \frac{v^2}{r}$$

Equation 1

Where, a_c [m/s^2] is the centripetal acceleration, v [m/s] is the tangential velocity, and r [m] is the radius of the circular path.

By Newton's second law, F = ma, the magnitude of the corresponding centripetal force is given by:

$$F_c = m \ a_c = \frac{m \ v^2}{r}$$

Equation 2

Where, F_c [N] is the centripetal force that acts on mass m [kg]. Thus, the magnitude of the corresponding centripetal force, whether applied by the tension in a string or the force due to a spring, equals that of Equation 2.
The velocity v [m/s] may be determined by the time it takes the mass to move around its

circular path. The circumference of a circle, and thus the distance traveled by the mass, is $2\pi r$. If the mass does traverse this distance in a time interval Δt, then the corresponding velocity is given by:

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T}$$

Equation 3

Where, r [m] is the radius of the circular path and T [s] is the period of rotation (the time interval it takes to complete one rotation around the circle).

Further, making the substitution of the velocity from Equation 3 into Equation 2 yields:

$$F_c = \frac{m v^2}{r} = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r m}{T^2}$$

Equation 4

This gives an expression for the centripetal force in terms of experimentally determinable quantities.

EXPERIMENTAL PROCEDURE

String & Mass:

The equipment for this section of the laboratory consists of masses (metal nuts) attached to one end of a string and a wooden mass attached to the other end of the string. The string is threaded through a covered glass tube so that tube may slide between the two ends of the string. The wooden mass will be swung in a circle, thus, creating a centripetal force on the stopper that will be balanced by the hanging masses (balancing force = total hanging mass * acceleration due to gravity). As noted earlier, the magnitude of the rotation determines the magnitude centripetal force and thus the amount of weight needed to balance the system.

Below is an illustration of this set-up.

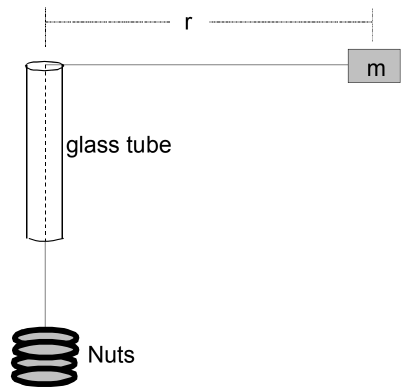

Figure 3

a) Before you begin taking data, practice rotating the stopper in a constant horizontal circular path.

- Does the rate at which you rotate the stopper determine whether it moves away from the tube, stays at a constant distance, or moves toward the tube? Explain.

A small piece of tape placed on the string at the base of the tube will give you a reference point to maintain a constant radial distance. Be careful not to hit yourself or anyone else in the head while rotating the wooden mass.

b) Place enough nuts on the string such that you are able to achieve a uniform circular path with the wooden mass.
c) Record the starting radius that you will be using. The radius distance is measured from the tube's mouth to the center of the wooden mass. The mass of the wood and the nuts will be useful as well.
d) Begin rotating the wooden mass at the assigned radius. Determine the period of rotation that gives the system a steady motion (mass not moving toward or away from the tube). The period is best determined by measuring the time for a number of revolutions (25 or more), then dividing the total time by the number of revolutions occurring over that time interval. Record the number of rotations counted and the time interval measured.

- What observations can you make about the path the mass follows while traveling around the center?
  - Does the string/mass remain level with the ground the entire time? Why should it or why should it not?
  - What affect would a non-horizontal circular path have on the results?

e) Repeat your observations for b-d for several additional trials, adding one or two nuts to the string for each successive trial.

Get a ✔CHECK from the instructor before proceeding

- Which force is responsible for balancing the centripetal force that the rotating stopper has?
    - Calculate the balancing force for your trials.
    - Calculate the centripetal force for each corresponding trial
        - How does the balancing force compare to the centripetal force?
        - Should they be in agreement or not?
- Experimentally, how does the centripetal force change by changing the balancing mass?
    - Is that conclusion supported by the equations that govern the centripetal force?
- How does the centripetal force change by changing the radius of the circular path?
    - Is that conclusion supported by the equations that govern the centripetal force?

Get a ✔CHECK from the instructor before proceeding

Equation (2) can be written as: $$v^2 = \left(\frac{1}{m}\right) F_c \; r$$

For a constant value of m this implies that $\mathbf{v^2}$ is proportional to $\mathbf{F_c r}$.

- Define each of the variables in the equation above. Be sure to indicate their units as well as how you'd determine them in the context of the laboratory.
- The equation above is a linear equation of the form: y = mx +b.
    - Which of the variables is plotted on the y-axis?
    - Which of the variables is plotted on the x-axis?
    - Which of the variables corresponds to the y-intercept?
    - Which of the variables corresponds to the slope?
- Using your data, make a graph of this equation.
- From a best-fit-straight-line of your data points, determine the y-intercept and the slope.
    - How do these values correspond to what you indicated that they should be equivalent to?

Get a ✔CHECK from the instructor before proceeding

Spring Apparatus:
The equipment for this section of the laboratory consists of a commercial centripetal force apparatus. This particular apparatus applies a balancing force via a spring. Below is an illustration of this device.

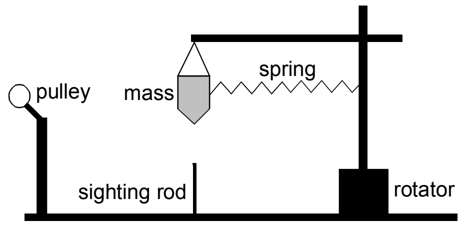

Figure 4

The device works in the same basic way that the string & stopper device works. The rotator shaft spins around causing the mass (bob) to experience a centripetal force that is supplied, in this case, by a spring. The sighting rod gives a radius starting location from which to work. The sighting rod can be moved to different linear positions in order to vary the radius of the circular path. Thus, each revolution of the mass should pass the bob directly over the sighting rod. This ensures a constant radius. This radius is measured from the center of the sighting rod to the center of the rotator shaft.

- For springs, what TWO things determine how far it will stretch if a force is applied?
    - We can determine one of these by attaching a string to the pulley side of the mass, draping the string over the pulley, and applying masses to the string until the spring stretches to the radii position used (the point directly over the sighting rod).
        - How would one determine the force that is causing the spring to stretch in this setup?
            - How does this compare to the force the spring is pulling back with?
            - If the apparatus was allowed to spin and the spring stretched out over-top of the sighting rod as before, how would the centripetal force of the spring compare to that of the applied force when it was stationary?

Get a ✔CHECK from the instructor before proceeding

a) Adjust the sighting rod of the system to a desired distance and begin rotating the system by twirling the rotator shaft between your thumb and index finger. Rotate at a rate such as to cause the bob to pass directly over the sighting rod with each revolution.
b) Once a steady rotation rate has been achieved, measure the period of revolution of the bob. Again, the period is best determined by measuring the time for a number of revolutions (25 or more), then dividing the total time by the number of revolutions occurring over that time interval.

c) With the system not rotating, attach a string to the side of the bob and drape it over the pulley. Attach a slotted mass hanger to the string's end draped over the pulley. Begin applying masses to the hanger, causing the spring to stretch. Add masses until the bob is directly over the sighting rod.

- What pieces of information would be needed to determine the centripetal force exerted by the bob on the spring while it was rotating?
    - Identify these quantities and compute the centripetal force from one of your observations.
        - How does this value compare to the force needed to stretch the non-rotation bob and spring?

Get a ✔CHECK from the instructor before proceeding

d) Repeat the observations and any data recording for the same radii but where a 100 g mass is added to the top of the bob.

- Experimentally, how does the centripetal force change by changing the mass of the rotating bob?
    - Is that conclusion supported by the equations that govern the centripetal force?

i) Repeat the observations from above for one additional sighting rod location.

- How does the centripetal force change by changing the radius of the circular path?
    - Is that conclusion supported by the equations that govern the centripetal force?
- Suppose a different spring, with a larger spring constant, was used. If you attempted to rotate a given bob mass, at one of the radii that was previously studied, how would the new period of rotation compare to the old spring's period of rotation?

Get a ✔CHECK from the instructor before concluding