

Centripetal Force - Traditional

OBJECTIVE

To verify that a mass moving in circular motion experiences a force directed toward the center of its circular path. To determine how the mass, velocity, and radius affect a particle's centripetal force. To explain why the centripetal force is necessary for circular motion.

INTRODUCTION

When any mass m moves in a circle of radius r with a constant velocity v , it experiences a centripetal force directed toward the center of its circular path. For this laboratory, we will study the case of uniform circular motion, where the speed (a scalar quantity) of the moving mass is held constant. Yet, as the mass moves in a circular path, the velocity (a vector quantity) is constantly changing. By definition a velocity that changes over time is an acceleration. Thus, this change in velocity comes from a centripetal acceleration due to a centripetal force. The centripetal force is that force that pulls an object out of a straight-line path and into a circular path. In this laboratory, the centripetal force will be provided by gravity and a spring (in separate cases). By measuring the frequency of rotation, the object's mass, path radius and the spring force, the centripetal force can be experimentally determined.

APPARATUS

(1) spring centripetal force apparatus, (1) meter stick, (1) slotted mass set, (1) stopwatch, (1) balance, (1) string & stopper centripetal force apparatus, and (10) washers.

THEORY

As previously stated, when a particle moves in a circular path, its velocity is constantly changing. The velocity of the particle is tangent to the path of its motion at all times. Thus, as it is ever changing its path, it is ever changing its velocity. The magnitude of the velocity, however, will remain constant. Note, the speed (velocity magnitude) does not have to be constant but would present further study to pursue what amounts to a changing centripetal force.

The figure below illustrates this motion.

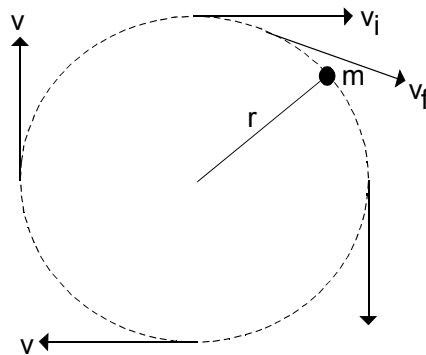


Figure 1

Note, if the mass were to break from the confines of its circular path, it would move off in a direction that is tangent to the circle at the point of release. Two of the tangential velocities have been intentionally labeled v_i and v_f to illustrate the direction of the resultant velocity. This is done by applying the relation for change in velocity: $\Delta v = v_f - v_i$. From the two labeled vectors in Figure 1 and the previously cited equation, we get:

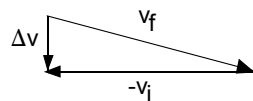


Figure 2

Note that Δv points toward the center of the circular path. This vector direction, for the velocity, will always be the same as long as we take v_i and v_f to have a very short time interval between them. This change in velocity results from a centripetal acceleration. It can be further shown that, the magnitude of this centripetal acceleration is given by:

$$a_c = \frac{v^2}{r}$$

Equation 1

Where, a_c [m/s^2] is the centripetal acceleration, v [m/s] is the tangential velocity, and r [m] is the radius of the circular path.

By Newton's second law, $F = ma$, the magnitude of the corresponding centripetal force is given by:

$$F_c = m a_c = \frac{m v^2}{r}$$

Equation 2

Where, F_c [N] is the centripetal force that acts on mass m [kg]. Thus, the magnitude of the corresponding centripetal force, whether applied by the tension in a string or the force due to a spring, equals that of Equation 2.

The velocity v [m/s] may be determined by the time it takes the mass to move around its circular path. The circumference of a circle, and thus the distance traveled by the mass, is $2\pi r$. If the mass does traverse this distance in a time interval Δt , then the corresponding velocity is given by:

$$v = \frac{\Delta x}{\Delta t} = \frac{2 \pi r}{T}$$

Equation 3

Where, r [m] is the radius of the circular path and T [s] is the period of rotation (the time interval it takes to complete one rotation around the circle).

Further, making the substitution of the velocity from Equation 3 into Equation 2 yields:

$$F_c = \frac{m v^2}{r} = \frac{m \left(\frac{2 \pi r}{T} \right)^2}{r} = \frac{4 \pi^2 r m}{T^2}$$

Equation 4

This gives an expression for the centripetal force in terms of experimentally determinable quantities.

EXPERIMENTAL PROCEDURE

String & Stopper:

The equipment for this section of the laboratory consists of masses (metal washers) attached to one end of a string and a rubber stopper attached to the other end of the string. The string is threaded through a covered glass tube so that tube may slide between the two ends of the string. The rubber stopper will be swung in a circle, thus, creating a centripetal force on the stopper that will be balanced by the hanging masses (balancing force = total hanging mass * acceleration due to gravity). As noted earlier, the magnitude of the rotation determines the magnitude centripetal force and thus the amount of weight needed to balance the system.

Below is an illustration of this set-up.

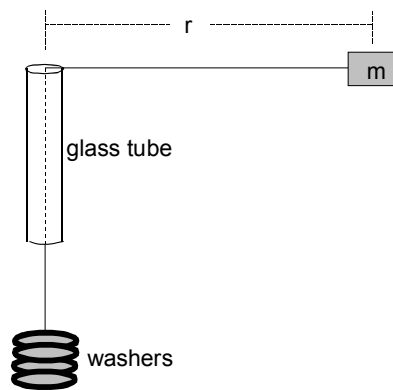


Figure 3

- a) Before you begin taking data, practice rotating the stopper in a constant circular path. You will notice that the rate at which you rotate the stopper determines whether it moves away from the tube, stays at a constant distance, or moves toward the tube. A small piece of tape placed on the string at the base of the tube will give you a reference point to maintain a constant radial distance. Be careful not to hit yourself or anyone else in the head while rotating the stopper.
- b) Place enough washers on the string such that you are able to achieve a uniform circular path with the rubber stopper. With the use of the balance, determine the mass of the rubber stopper and the mass of the washers used. Record these values in the data table.
- c) Record the starting radius that you will be using. The radius distance is measured from the tube's mouth to the center of the rubber stopper.
- d) Begin rotating the stopper at the assigned radius. Determine the period of rotation that gives the system a steady motion (stopper not moving toward or away from the tube). The period is best determined by measuring the time for a number of revolutions (25 or more), then dividing the total time by the number of revolutions occurring over that time interval. Record the number of rotations counted and the time interval measured.
- e) Repeat b-d for four additional trials, adding one or two washers to the string for each successive trial.

Spring Apparatus:

The equipment for this section of the laboratory consists of a commercial centripetal force apparatus. This particular apparatus applies a balancing force via a spring. Below is an illustration of this device.

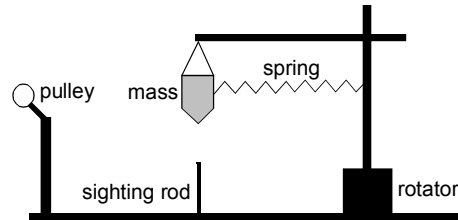


Figure 4

The device works in the same basic way that the string & stopper device works. The rotator shaft spins around causing the mass (bob) to experience a centripetal force that is supplied, in this case, by a spring. The sighting rod gives a radii location from which to work. The sighting rod can be moved to different linear positions in order to vary the radius of the circular path. Thus, each revolution of the mass should pass the bob directly over the sighting rod. This ensures a constant radius. This radius is measured from the center of the sighting rod to the center of the rotator shaft.

The force that the spring exerts on the bob is proportional to the amount of the spring that is stretched. The force can be determined experimentally by the use of the pulley that is a part of the device. This is accomplished by attaching a string to the pulley side of the mass, draping the string over the pulley, and applying masses to the string until the spring stretches to the radii position used (the point directly over the sighting rod). This should be done for each new radius studied. This mass times the acceleration due to gravity is the experimental value of the centripetal force provided by the spring.

- a) Detach the bob from the hanger strings and the spring. With the use of a laboratory balance, determine its mass. Record this value in the data table.
- b) Reattach the bob to the hanger strings but not the spring. Adjust the position of the sighting rod to its closest position to the rotator shaft. Loosen the screw atop the rotator shaft, that holds the support arm for the bob, and adjust the arm so that the bob hangs directly over the sighting rod. Tighten the screw back and reattach the spring to the bob.
- c) Measure and record the distance from the center of the sighting rod to the center of the rotator shaft.
- d) Begin rotating the system by twirling the rotator shaft between your thumb and index finger. Rotate at a rate such as to cause the bob to pass directly over the sighting rod with each revolution.
- e) Once a steady rotation rate has been achieved, measure the period of revolution of the bob. Again, the period is best determined by measuring the time for a number of revolutions (25 or more), then dividing the total time by the number of revolutions occurring over that time interval. Record the number of rotations counted and the time interval measured.

- f) Repeat d and e twice more for the same radii and bob mass. Be sure to count the same number of revolutions as was done in part e for each of these two trials. The objective is to obtain an average time measure for computing the period of revolution.
- g) With the system not rotating, attach a string to the side of the bob and drape it over the pulley. Attach a slotted mass hanger to the string's end draped over the pulley. Begin applying masses to the hanger, causing the spring to stretch. Add masses until the bob is directly over the sighting rod. Record this mass as the experimental mass equivalent to the spring's centripetal force.
- h) Repeat d - g for the same radii but for two different bob masses. One where a 50 g slotted mass is added to the top of the bob and a second where a 100 g mass is added to the top of the bob.
- i) Repeat b - h for two additional radii of rotation. One where the sighting rod is placed at the mid-point of its two extremes and a second where the sighting rod is placed at a point that is its maximum distance from the rotator shaft.

CALCULATIONS

String & Stopper: (Repeat for each of the five trials)

- Calculate the balancing force (experimental centripetal force) by multiplying the washer mass by the acceleration due to gravity.
- Divide the time interval measured by the number of revolutions counted in that time interval, to determine the period of one rotation.
- Using equation (3), determine the average experimental tangential velocity of the system.
- Using equation (4), where m [kg] is the stopper's mass, calculate the theoretical centripetal force.
- Calculate the percentage difference between the experimental and theoretical centripetal forces.

Spring Apparatus:

- Calculate the average time for the three trials of each case studied.
- Determine the period by dividing the average time interval measured by the number of revolutions counted in that time interval.
- Using equation (4), calculate the theoretical centripetal force. Be sure to use the total bob mass used in each calculation, where, in some cases the bob mass was increased by the addition of slotted masses.
- Determine the experimental centripetal force by multiplying the mass that was used to stretch the spring by the acceleration due to gravity.
- Calculate the percentage difference between the experimental and theoretical centripetal forces.

GRAPHS

String & Stopper: Equation (2) can be written as: $v^2 = \left(\frac{l}{m} \right) F_c r$

For a constant value of m this implies that v^2 is proportional to $F_c r$. The values for v^2 come from the computed tangential velocity from *Calculations, Part c*. The values for F_c come from *Calculations, Part a*. Plot v^2 on the y-axis and $F_c r$ on the x-axis for each of the five trials using your experimental data. Perform a least squares fit calculation to determine the slope and y-intercept of the plotted data. Record the values of the slope and y-intercept for future analysis. Indicate the best-fit-straight-line on your graph.

Spring Apparatus: NONE

DATA TABLE LAYOUT

Table #1: Constants:
Mass of Stopper [kg] and Mass of Bob [kg].

String & Stopper: Table #2: Gravitational Centripetal Force:
(For each of the five trials)
Mass of Washers [kg], Radius of Orbit [m], Number of Revolutions Counted [#], Time for Total Revolutions [s], Period [s], Experimental Tangential Velocity [m/s], Experimental Centripetal Force [N], Theoretical Centripetal Force [N], and Percentage Difference [%].

Spring Apparatus: Table #3: Spring Centripetal Force:
Total Mass of Bob [kg], Radius of Orbit [m], Number of Revolutions Counted [#], Time for Total Revolutions #1 [s], Time for Total Revolutions #2 [s], Time for Total Revolutions #3 [s], Experimental Mass Needed to Stretch Spring [kg], Average Time for Total Revolutions [s], Period [s], Experimental Centripetal Force [N], Theoretical Centripetal Force [N], and Percentage Difference [%].

POST-LABORATORY QUESTIONS

1) Experimentally, how does the centripetal force change by changing the mass? The radius of the circular path?

2) In the *String & Stopper* section of the laboratory, was the circle that the rubber stopper "made" perpendicular to the tube? Meaning, was a uniform horizontal circle swept out? What effect would a non-horizontal circular path have on the results?

3) From your graph of v^2 vs. $F_c r$ the slope determination from least squares should be equal to the inverse of the rubber stopper's mass. Additionally, the y-intercept should have been zero. How do these values compare to what you actually recorded? Explain any differences.

4) Suppose in the *Spring Apparatus* section of the laboratory an additional spring, with a larger spring constant, was used. If you attempted to rotate a given bob mass, at one of the radii that was previously studied, how would the new period of rotation compare to the old spring's period of rotation?