Torque

OBJECTIVE

To verify the rotational and translational conditions for equilibrium. To determine the center of gravity of a rigid body (meter stick). To apply the torque concept to the determination of an unknown mass.

INTRODUCTION

When an unbalanced force acts on a body, the body has the tendency to rotate (about some axis), translate, or both. For a body to be in equilibrium, it must have both rotational equilibrium (sum of all the torques is equal to zero) and translational equilibrium (sum of all linear forces is equal to zero). The total equilibrium condition is called static equilibrium. In this laboratory, a meter stick will serve as a rigid body to which forces (masses under the influence of gravity) act. From these applied forces and the torque, the concept of center of gravity and center of mass will be investigated.

APPARATUS

(1) meter stick, (1) meter stick support stand, (1) balance, (4) string loops, (1) knife-edge clamp without hanger loop, (1) hooked mass set, and
(1) unknown mass with hook.

THEORY

Torque: When a force acts on a rigid body, that is allowed to pivot about some axis, the body will have the tendency to rotate. The tendency toward rotation is called torque, τ. The torque is a result of a force F that is applied at a distance d_ζ that is perpendicular to the applied force extending from the axis of rotation. The equation is thus given as:

$$\tau = F_\perp \, d$$

Equation 1

Where, τ [N m] is the torque, F [N] is the applied force, and d_ζ [m] is the perpendicular distance. Note, both d_ζ and F are vectors as well as the torque itself.

To fully understand the concept of the perpendicular distance, let us illustrate an applied force that is present on a rigid body.

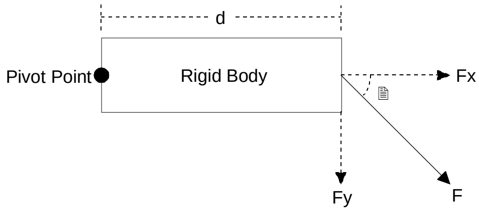

Figure 1

In Figure 8-1, d is the distance from the pivot point to the force F. As mentioned previously, only the applied force that is perpendicular to the distance applies the torque (rotation). Thus, based on the figure, only the y-component of the applied force contributes to the rotation of the body. In equation form this is expressed as:

$$\tau = (F \sin\theta)\, d$$

Equation 2

Which is the same form as equation (1) if $\theta = 90^o$. This will generally be the case for torques. That is, the applied force will automatically be perpendicular to the distance.

This force component can either cause the rotation, relative to the axis of rotation, to be in a clockwise (CW) or counterclockwise (CCW) direction. As only a rotational motion is established (non-linear), the vector direction for the rotation is set so that CW motion is designated as the negative direction and CCW motion is designated as the positive direction.

Equilibrium: The conditions for static equilibrium, as stated previously, are that the vector sum of the forces must equal zero and that the vector sum of the torques must equal zero.

$$\sum F = 0 \quad and \quad \sum \tau = 0$$

Equation 3

The vector sum of the forces is equal to zero is concerned with the translational (linear) equilibrium of the rigid body. That is, the object is not moving in any of the possible linear directions, whether with a linear acceleration or with constant linear velocity. As the laboratory will make use of a meter stick as the rigid body, it is restricted from linear motion, and thus, this condition is automatically satisfied. Even if an object is not moving in a linear direction, it can still be "moving", i.e. have a rotation. The vector sum of the torques equal to zero is concerned with the rotational equilibrium of the rigid body. That is, the object is not moving in either a CW or CCW direction, whether with an angular acceleration or with constant angular velocity.

It is this vector sum of the torques that will be the concentration factor in this laboratory. More precisely, the sum of the torques in the CW direction should equal the sum of the torques in the CCW direction, if the object is to be in rotational equilibrium.

$$\sum \tau_{CW} - \sum \tau_{CCW} = 0$$

Equation 4

Let us refer to Figure 2, below, to develop and an understanding of Equation 4.

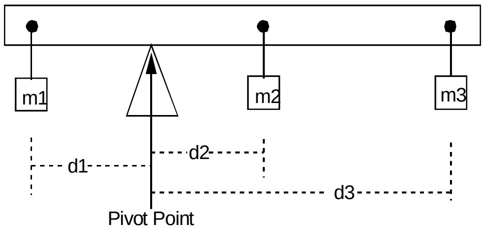

Figure 2

Where, m_1, m_2 and m_3 are the respective masses that are applying the force to the rigid body (due to the influence of gravity) and d_1, d_2, and d_3 are the distances from the pivot point to their respective applied forces (masses); these distances are called lever arm distances.

Note, mass m_1 provides a CCW torque to the object and masses m_2 and m_3 provide the CW torque to the object. Thus, in the sum of the torques, the applied force due to m_2 and the applied force due to m_3 act in the same rotational direction and therefore add. The respective torques, on the object, follows:

$$\tau_1 = F_1 \; d_1 = m_1 \; g \; d_1$$
$$\tau_2 = F_2 \; d_2 = m_2 \; g \; d_2$$
$$\tau_3 = F_3 \; d_3 = m_3 \; g \; d_3$$

Equation 5

Applying Equation 5 to Equation 4 yields:

$$\tau_2 + \tau_3 = \tau_1$$
$$or$$
$$F_2 \; d_2 + F_3 \; d_3 = F_1 \; d_1$$
$$or$$
$$m_2 \; g \; d_2 + m_3 \; g \; d_3 = m_1 \; g \; d_1$$
$$or$$
$$m_2 \; d_2 + m_3 \; d_3 = m_1 \; d_1$$

Equation 6

Note, in the last line of Equation 6, the acceleration due to gravity was canceled out as it appears in all terms and is, therefore, unnecessary for the computation. This form of the torque equation will be used as the basis for future calculations in this laboratory.
Note, the absence of the acceleration due to gravity in Equation 6 does allow for the computation of unknown masses and/or distances but does not result in the torque of the system. The torque has units of Newton-meters which do not result from the product of mass times distance.

The Center of Gravity: The center of gravity of an object is defined by the point on which the sum of the gravitational torques, due to the "individual" mass particles in the object, is equal to zero. By "individual" mass particles, we mean that a rigid object is considered to be made up of tiny bits of mass. The combination of each of these masses makes up and describes the rigid body. For a uniform rigid body (same density throughout), each particle contributes to the torque about a pivot point. Again, each of these torques is due to the mass particles acting under the influence of gravity; thus, the applied force is actually weight. The idea is to locate the point on the object where the total weight of the object is concentrated, and thus the location where the effect on the rotation of the object is the same as that of the individual particles.

The object is in equilibrium when it is supported by a force equal to its weight, and the sum of the gravitational forces acting on the individual masses about the center of gravity equal zero.

In a symmetric object (like a meter stick) the center of gravity is located on the symmetry axis of the object. Given a meter stick, the center of gravity is located at the 50.0 cm mark. Supporting the meter stick here results in no rotation of the object. However, venture from this mark and the meter stick will rotate as the sum of the torques CW does not equal the sum of the torques CCW.

Gravitational forces do act on the rigid body as a whole. Thus, the location (for a uniform rigid body) of the weight concentration is also the location of the mass concentration in the object. Thus, we can refer to the center of gravity as the center of mass as long as the acceleration due to gravity is constant throughout the object.

PRELIMINARY QUESTIONS

These questions should be completed prior to coming to lab. The answers will involve you not only reading the laboratory theory section but recalling the associated lecture material. Please complete these questions on a separate sheet of paper in order that they may be collected at the beginning of lab.

1) What are the conditions for equilibrium of a rigid body?

2) What is the definition of torque? State the equation and define the variables.

3) Define the center of gravity of a rigid body? How does this compare to the center of mass?

4) What is meant by clockwise and counterclockwise torques? How are these applied in the context of this laboratory?

EXPERIMENTAL PROCEDURE

a) Determine the mass of the meter stick (m_g). A laboratory balance is provided for this determination. Record the meter stick mass in your data table.

b) Slide the unhooked hanger on the meter stick and clamp it near the center of the stick. Place the meter stick-hanger combination on the support stand. Adjust the meter stick from side-to-side, through the hanger, until the meter stick hangs level (in equilibrium). Tighten the clamp to the meter stick at this point and record this clamped position as x_g in your data table. This is the location of the center of gravity of your meter stick.

Condition 1: Two Known Masses:
a) With the meter stick pivoted at x_g, place a 100 g mass at the 10 cm position on the meter stick (10 cm from zero).
b) Place a 200 g mass on the opposite end of the meter stick at a position that causes the system to be in static equilibrium.
c) Record the two masses and the two positions in the data table.
d) Repeat for a 100 g mass at 25 cm and the 200 g mass at a new equilibrium point.

Condition 2: Three Known Masses:
a) With the meter stick pivoted at x_g, place a 100 g mass at the 10 cm mark and a 200 g mass at the 68 cm mark on the meter stick.
b) Place a 50 g mass at a position in the system that causes the system to be in static equilibrium.
c) Record the three masses and their respective positions in the data table.
d) Repeat for balancing the system with a 100 g mass used to place the system in static equilibrium.

Condition 3: Meter Stick Mass:
a) Place a 100 g mass at the 10 cm mark on the meter stick.
b) Loosen the clamp that holds the meter stick at x_g. Slide the meter stick in the clamp until you are able to place the system in static equilibrium.
c) Record the mass, its position, and the location of the clamp that provided the static equilibrium position.
d) Repeat for a 50 g mass placed at 10 cm.

Condition 4: Meter Stick Center of Gravity:
a) Place the clamp, on the meter stick, at the 40 cm mark.
b) Place a 50 g mass between 5-10 cm, a 200 g mass at 30 cm, and a 100 g mass at 90 cm.
c) Determine the position at which a 100 g mass should be placed to cause the system to be in static equilibrium.
d) Repeat for the meter stick clamped at the 55 cm mark.

Condition 5: Unknown Mass:
a) By the use of any of the previously outline procedures, determine the mass of the unknown provided.
b) Record all necessary values (masses, positions, and the pivot point) in the data table. Be sure to record the identification number/letter of your unknown in the data table.
c) Repeat the above instructions for a different experimental condition.

CALCULATIONS

a) The distances to be used for the torque calculations are called lever arm distances. As previously stated, the distance from the pivot point to each force (hanging mass * gravity) is the lever arm distance. In each case determine the lever arm distance by subtracting the pivot point location from the mass locations. Be sure that you record this value as a positive number (a magnitude). You want to have the actual distance between the two points.

Condition 1: Two Known Masses:
a) Using the masses and the corresponding lever arm distances, compute the CW and CCW torques for each of the two cases.
b) Calculate the percentage difference between the CW and CCW torques for each of the two cases.

Condition 2: Three Known Masses:
a) Using the masses and the corresponding lever arm distances, compute the CW and CCW torques for each of the two cases.
b) Calculate the percentage difference between the CW and CCW torques for each of the two cases.

Condition 3: Meter Stick Mass:
a) If not previously computed, determine the lever arm distance for where the meter stick mass is located. This is accomplished by subtracting the pivot point from x_g.
b) Calculate the respective torques due to the 100 g mass and the 50 g mass.
c) The torque on the opposite of the pivot point is due to the mass of the meter stick acting under the influence of gravity.
d) Calculate the percentage difference between the known mass and the experimental mass of the meter stick, for both the 100 g case and the 50 g case.

Condition 4: Meter Stick Center of Gravity:
a) Using the masses and the corresponding lever arm distances, compute the CW and CCW torques for each of the two cases.
b) Calculate the percentage difference between the CW and CCW torques for each of the two cases.

Condition 5: Unknown Mass:
a) Calculate the value of the unknown mass for each of the two experimental arrangements.

GRAPHS

NONE calculated, however, draw (sketch) a diagram of each of the conditions and cases studied. Label all parts of each sketch, including the associated values of each mass, position, and lever arm distance. Figure 2 is a sample representation of what your sketches should look like.

DATA TABLE LAYOUT - Suggested

Table #1: Constants:
Meter Stick Mass [kg] and Meter Stick Center of Gravity [m]

Condition 1: Table #2: *Two Known Masses:*
Mass #1 [kg], Position #1 [m], Lever Arm #1 [m], Mass #2 [kg], Position #2 [m], Lever Arm #2 [m], CW Torque [Nm], CCW Torque [Nm], and Percentage Difference [%].

Condition 2: Table #3: *Three Known Masses:*
Mass #1 [kg], Position #1 [m], Lever Arm #1 [m], Mass #2 [kg], Position #2 [m], Lever Arm #2 [m], Mass #3 [kg], Position #3 [m], Lever Arm #3 [m], CW Torque [Nm], CCW Torque [Nm], and Percentage Difference [%].

Condition 3: Table #4: *Meter Stick Mass:*
Mass #1 [kg], Position #1 [m], Lever Arm #1 [m], Pivot Point [m], Meter Stick Lever Arm [m], Meter Stick Mass Experimental [kg], and Percentage Difference [%].

Condition 4: Table #5: *Meter Stick Center of Gravity:*
Pivot Location [m], Mass #1 [kg], Position #1 [m], Lever Arm #1 [m], Mass #2 [kg], Position #2 [m], Lever Arm #2 [m], Mass #3 [kg], Position #3 [m], Lever Arm #3 [m], Meter Stick Mass [kg], Meter Stick Lever Arm [m], Mass #4 [kg], Position #4 [m], Lever Arm #4 [m], CW Torque [Nm], CCW Torque [Nm], and Percentage Difference [%].

Condition 5: Table #6: *Unknown Mass:*
Method # [#], Mass(es) [kg], Position(s) [m], Lever Arm(s) [m], and Experimental Unknown Mass [kg].

POST-LABORATORY QUESTIONS

1) In most cases x_g, the center of gravity of the meter stick, is not located at the 50.0 cm mark. What would cause the position of x_g to not be located at 50.0 cm?

2) For Condition 1, explain how Equation 3 is satisfied.

3) Compared to Conditions 1 & 2, Conditions 3, 4, & 5 had the pivot point of the meter stick located at a position other than its center of gravity. Why is the mass of the meter stick included in the calculations when the pivot point is not located at x_g? Why is the mass of the meter stick not included in the calculations when the pivot point is located at x_g?

4) According to the definition of torque, our calculations should have included a force time a distance. However, we only used the masses present during the laboratory and not any force. Why was this not considered to be a "big deal;" i.e. why were the masses used in favor of the forces and why did it not matter?